- Session 53: Animal Genetics – methodology
  - rcantet@agro.uba.ar

Estimating the covariance structure for environmental effects in weaning weight of beef cattle

R.J.C.Cantet,[1, 2*] A. N. Birchmeier, [1]

1. Dep. Producción Animal, FA UBA; 2. CONICET

ARGENTINA

Introduction

- The magnitude of the direct-maternal additive covariance (σ_{AoAm}) for weaning weight in beef cattle is still an issue of debate.

- A possible reason for the estimated values of σ_{AoAm} (r_{AoAm}) is the presence of a covariance between direct and maternal environmental effects (σ_{EoEm} Koch, 1972), which is present in cov (Offspring, Dam).

- "Maternal environment for gain from birth to weaning seems to be significantly and negatively affected by direct effects of *maternal environment from previous generations*. Speculation suggests a value of –0.1 to –0.2 for this direct path", Koch (1972).

Environmental covariance (σ_{EoEm})

Model of Falconer (1965) (*regression on maternal phenotype*): Cantet *et al* (1988), Koerhuis and Thompson (1997), Meyer (1997).

However, Bijma (2006) observed that inheritance in Falconer's regression model is no longer Mendelian, and depends on the regression coefficient of the maternal phenotype.

- Quintanilla *et al* (1998) proposed a covariance-structure among permanent environmental effects that accounts for σ_{EoEm} in cov(O,D). However, it only shows in the covariance *between a dam and her offspring dam. However*, σ_{EoEm} does not arise in the covariance among dams and male calves, or dams and female calves that do not become dams, as in Koch's formulation.

Objectives

1) To estimate σ_{EoEm} (parametrized as a correlation, ρ), for weaning weight of Brangus and Hereford calves using Bayesian methods.

2) To compare the estimates of σ_{AoAm} (r_{AoAm}) from models that include or not ρ, and an informative covariance structure for environmental effects.

- Subset from 6 herds of the genetic evaluation program (ERBra) of Argentine Brangus Association, and a Hereford purebred herd.

	Weaning weights	Dams	Animals in pedigree
Brangus	1943	967	3222
Hereford	5503	2017	6860

- All dams with records have their dams known.
- Brangus: data pre-corrected for solutions from ERBra 2008; fixed effects in the model were age of calf (linear covariate), sex, and age of dam. Hereford: direct analysis.

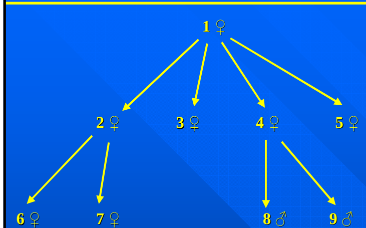
Model

$$y = X\beta + Z_o a_o + Z_m a_m + \varepsilon + e$$

- $E(y) = X\beta$ $[a_o \ a_m]' \sim N(0, G_0 \otimes A)$ $e \sim N(0, I\sigma^2_e)$
  - ε = random ***environmental individual effects*** : $\varepsilon \sim N(0, \Sigma_\rho)$
- “The amount of "permanent" environmental variation included will vary depending on whether maternal half sibs were adjacent or separated by 2 or more years”.

Koch (1972).

Σ_ρ for a family



$$r_{ij} = 1 - \frac{\text{abs}\left(\begin{array}{l}\text{Julian birth date } i - \\ \text{Julian birth date } j\end{array}\right)}{\text{Max. difference for dam of } i \text{ and } j}$$

Cow 2 is the dam of calf 7

$$\Sigma_\rho = \begin{bmatrix} 1 & \rho & \rho & \rho & \rho & 0 & 0 & 0 & \\ \rho & 1 & r_{23} & r_{24} & r_{25} & \rho & \rho & & 0 \\ \rho & r_{32} & 1 & r_{34} & r_{35} & 0 & 0 & 0 & 0 \\ \rho & r_{42} & r_{43} & 1 & r_{45} & 0 & 0 & \rho & \rho \\ \rho & r_{52} & r_{53} & r_{54} & 1 & 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 & 0 & 1 & r_{67} & r_{68} & r_{69} \\ 0 & \rho & 0 & 0 & 0 & r_{76} & 1 & r_{78} & r_{79} \\ 0 & 0 & 0 & \rho & 0 & r_{86} & r_{87} & 1 & r_{89} \\ 0 & 0 & 0 & \rho & 0 & r_{96} & r_{97} & r_{98} & 1 \end{bmatrix} \sigma_\varepsilon^2$$

Bayesian analysis: prior distributions

- ***Fixed effects***: N (**0**, ***K***), diagonal cov-matrix ***K*** (K_i >10 [8]) ("proper prior": Hobert and Casella, 1996).
- ***Breeding values***: $[\boldsymbol{a}_o\ \boldsymbol{a}_m]' \sim N(\boldsymbol{0}, \boldsymbol{G}_0 \otimes \boldsymbol{A})$
- ***Covariance matrix of breeding values:*** **Inverted Wishart**
- ***Variances of environmental effects and error***: **scaled inverted chi-square densities.**
- ρ - ***parameter***: **Uniform, such that Σ_ρ is p. d.** Gibbs sampling of ρ as in Heringstad *et al* (2003) *J. Dairy Sci*. 86 : 653-660.

Sampling of ρ

- Heringstad *et al* (2003) *J. Dairy Sci*. 86 : 653-660

  Reorder and partition $\varepsilon = [\varepsilon_D'\ \varepsilon_P']$ (D = dam, P = progeny) so that

$$p(\varepsilon_D, \varepsilon_P \mid \boldsymbol{P}) \sim N_n(\boldsymbol{0}, \boldsymbol{P}\sigma^2_\varepsilon)$$

<u>*Regression model*</u>

$$\varepsilon_D = \rho\ Z_p\, \varepsilon_P + e^*$$

$$e^* \sim N_d(\boldsymbol{0}, \boldsymbol{I}\,\sigma^2_{e^*}) \qquad \sigma^2_{e^*} = (1-\rho^2)\,\sigma^2_\varepsilon$$

Conditional posterior density of ρ

$$p\,(\rho \mid \varepsilon_D, \varepsilon_P, \sigma^2_{e*}) \sim N\,(\mathbf{E}_c\,(\rho), \mathbf{Var}_c(\rho))$$

constrained such that Σ_ρ **is p.d.**

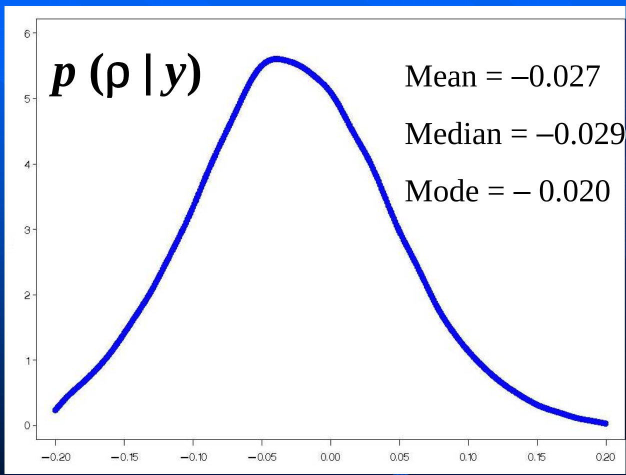
$$\mathbf{E}_c(\rho) = (\varepsilon_P{'}\mathbf{Z}_p\,\mathbf{P}^{-1}\,\mathbf{Z}_p\,\varepsilon_P)^{-1}\,\varepsilon_P{'}\mathbf{Z}_p\,\mathbf{P}^{-1}\,\varepsilon_D$$

$$\mathbf{Var}_c(\rho) = (\varepsilon_P{'}\mathbf{Z}_p\,\mathbf{P}^{-1}\,\mathbf{Z}_p\,\varepsilon_P)^{-1}\,\sigma^2_{e*}$$

Density of the regression error variance

$$p\,(\sigma^2_{e*} \mid \varepsilon_D, \varepsilon_P, \rho) \sim e^{*'}\,e^{*}\;\chi^{-2}_{d-2}$$

Brangus: Posterior distribution of ρ



Values of ρ

Brangus – Posterior means of parameters

Parameter	"Usual Model"	Model with Σ_ρ
σ^2_{Ao}	91.25	90.26
σ_{AoAm}	–22.25	–19.91
σ^2_{Am}	56.57	70.52
σ^2_{Em} or σ^2_ε	74.64	45.50
σ^2_e	475.60	474.11
ρ	–	–0.027
r_{AoAm}	–0.30	–0.23

Hereford - Posterior means of parameters

Parameter	"Usual Model"	Model with Σ_ρ
σ^2_{Ao}	143.96	77.97
σ_{AoAm}	–83.70	–51.09
σ^2_{Am}	137.93	156.02
σ^2_{Em} or σ^2_{ε}	158.33	216.41
σ^2_{e}	509.05	488.22
ρ	–	0.003
r_{AoAm}	–0.59	–0.46

Posterior correlation matrices for Σ_ρ model

Her \ Bra	σ^2_{Ao}	σ_{AoAm}	σ^2_{Am}	σ^2_{Em} or σ^2_{ε}	ρ	σ^2_{e}
σ^2_{Ao}		–0.50	0.06	–0.10	–.003	0.21
σ_{AoAm}	–0.63		–0.60	0.02	0.12	0.00
σ^2_{Am}	0.19	–0.71		–0.11	–0.12	–.003
σ^2_{Em} or σ^2_{ε}	0.14	–0.61	0.87		–0.15	–0.02
ρ	–0.49	0.44	–0.35	–0.33		.002
σ^2_{e}	0.98	–0.62	0.19	0.14	–0.49	

Conclusions

- The estimates of r_{AoAm} (σ_{AoAm}) from the model including Σ_ρ were less negative in both data sets, as compared with the estimates of r_{AoAm} from the classic model.

- Estimates of σ^2_{Ao} from both models were more similar in Brangus than in Hereford, due to differences in the amount of information for the parameters in the data.

- The environmental correlation among maternal half sibs varied depending on how distant were their birthdates, and this has probably more effect on the magnitude of r_{AoAm} than including ρ in the covariance structure.