# Data Transformation for Rank Reduction in Multi-Trait MACE models

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**Data Transformation for Rank Reduction in Multi-Trait MACE Models**

# The joint French-German evaluation

- **Improve international genetic evaluations**
  - **Multiple trait MACE models (MT-MACE)**
    **vs. Single trait MACE**
  - **Full pedigree information (sire-dam)**
    **vs. (Sire-MGS-MGD)**
  - **Pre-corrected records (DYDs and EDCs)**
    **vs. Deregressed proofs**

# The Multiple Trait MACE model (MT-MACE )

- **National evaluation to obtain pre-corrected records (q)**
  - **DYD: average of daughter performance adjusted for fixed effects and non-genetic random effects of daughters and genetic effects of bull's mates**

- **International evaluation model**

$$\mathbf{q_i} = \boldsymbol{\mu} + \mathbf{a_i} + \mathbf{e_i}$$

where $Var(\mathbf{a_i}) = \mathbf{G}$ and $[Var(\mathbf{e}_i)]^{-1} = \boldsymbol{\Psi}_i$

- **Mixed Model Equations for bull i**

$$\left[\boldsymbol{\psi_i} + a^{ii}\mathbf{G}^{-1}\right]\hat{\mathbf{a}}_\mathbf{i} - \frac{1}{2}d_i\mathbf{G}^{-1}\left(\hat{\mathbf{s}}_\mathbf{i} + \hat{\mathbf{d}}_\mathbf{i}\right) = \boldsymbol{\Delta_i}$$

# The MT-MACE model for milk yield

- **The Random Regression MACE model (RR-MACE)**
  - German RRTDM (3 RRC*3 Lactations) (n=9)
  - Each bull 9 DYDs & 9x9 EDCs
  - 14487 Holstein bulls with data
  - Estimated RRC are highly correlated with National RRTDM
  - Computation requirements can be a limiting factor (i.e. joint bull and cow evaluations) → Rank Reduction

- **The Multiple Lactation MACE model (ML-MACE)**
  - Convert RRC to 305-day lactation basis =

    305(1st RRC) + 0.0009(2nd RRC) + 2.24(3rd RRC)
  - Each bull 3 DYDs & 3x3 EDCs (3 Lactations) (n=3)

# ML-MACE vs RR-MACE

- **EBV on a combined lactation weight (mean of 3 lactations)**

| Birth year | No. of bulls | r |
|---|---|---|
| 1985-1989 | 3955 | 1.000 |
| 1990-1994 | 4088 | 1.000 |
| 1995-1999 | 5194 | 0.999 |
| 2000 | 867 | 0.980 |
| 2001 | 211 | 0.924 |
| <10 daughters past 120DIM | 172 | 0.840 |
| Overall | 14487 | 0.995 |

- **Ranking correlations for top 300 bulls: 0.85**

# Rank reduction (rG-MACE)

- **Decomposing German genetic (co)variance matrix (G)**

$$\mathbf{G} = \mathbf{S}\,\mathbf{rG}\,\mathbf{S}$$

- **Eigenvalues of the genetic correlation matrix (rG)**

**4.22 2.57 1.52 0.30 0.21 0.10 0.03 0.02 0.02**

- **Obtain the transformation matrix T using eigenvalues D and eigenvectors U of genetic correlation matrix**

$$\mathbf{T} = \mathbf{S}\,\mathbf{U}\,\mathbf{D}^{\frac{1}{2}}$$

- **Rank reduction of MME from 9 traits to r eigenfunctions**

$$\boldsymbol{\Psi}_{rG} = \mathbf{T}'\boldsymbol{\Psi}\mathbf{T} \qquad \boldsymbol{\Delta}_{rG} = \mathbf{T}'\boldsymbol{\Delta}$$

# Rank reduction to 6 eigenfunctions

- **Back transformed proofs were a=Ta***
- **Pearson correlations with RR-MACE**

| | | Lactation | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| Random regression coefficient | First | 1.000 | 1.000 | 0.999 |
| | Second | 0.998 | 0.997 | 0.998 |
| | Third | 0.991 | 0.991 | 0.992 |
| 305-day lactation basis | | 1.000 | 0.998 | 0.991 |
| Combined lactation basis | | | 1.000 | |

- **Ranking correlations for top 300 bulls: 0.98**

# Further rank reduction (CW-MACE)

- **Different weight of RRC to convert to lactation basis**

| 1st RRC | 2nd RRC | 3rd RRC |
|---|---|---|
| 305 | 0.0009 | 2.24 |

- **Multiply the German genetic (co)variance matrix G by the combined lactation weights W**

$$\mathbf{C} = \mathbf{W}\,\mathbf{G}\,\mathbf{W}$$

- **Eigenvalues of matrix C**

$$9.91 \quad 0.76 \quad 0.11 \quad 10^{-5} \quad 10^{-6} \quad 10^{-7} \quad 10^{-10} \quad 10^{-11} \quad 10^{-12}$$

- **Obtain the transformation matrix T using eigenvalues D and eigenvectors U of matrix C**

$$\mathbf{T} = \mathbf{W}^{-1}\mathbf{U}\,\mathbf{D}^{\frac{1}{2}}$$

# Rank reduction to 3 eigenfunctions (1)

- **Pearson correlations with RR-MACE**

| | Lactation | rG-MACE | CW-MACE |
|---|---|---|---|
| 1st RRC | 1 | 0.992 | 0.998 |
| | 2 | 0.963 | 0.984 |
| | 3 | 0.952 | 0.984 |
| 2nd RRC | 1 | 0.948 | 0.783 |
| | 2 | 0.946 | 0.645 |
| | 3 | 0.899 | 0.531 |
| 3rd RRC | 1 | 0.975 | 0.614 |
| | 2 | 0.960 | 0.571 |
| | 3 | 0.950 | 0.532 |

# Rank reduction to 3 eigenfunctions (2)

- **Pearson correlations with RR-MACE**

| Birth year | No. of bulls | rG-MACE | CW-MACE | ML-MACE |
|---|---|---|---|---|
| 1985-1989 | 3955 | 0.988 | 0.996 | 1.000 |
| 1990-1994 | 4088 | 0.985 | 0.995 | 1.000 |
| 1995-1999 | 5194 | 0.979 | 0.992 | 0.999 |
| 2000 | 867 | 0.963 | 0.992 | 0.980 |
| 2001 | 211 | 0.983 | 0.993 | 0.924 |
| <10 daughters past 120DIM | 172 | 0.985 | 0.989 | 0.840 |
| Overall | 14487 | 0.987 | 0.995 | 0.995 |

# Conclusions

- **Random Regression MACE and National evaluations are highly correlated**

- **Multiple Lactation MACE:**
  - **Low correlations for young bulls with short lactations of daughters**
- **Rank reduction from 9 RRC to 6 eigenfunctions of genetic correlation matrix:**
  - **Back transformed EBV highly correlated for the 9 RRC**
  - **EBV and rankings on a combined lactation basis equal to RR-MACE**
- **Rank reduction from 9 RRC to 3 eigenfunctions based on combined lactation weights:**
  - **Back transformation to 9 RRC not possible**
  - **High correlations with RR-MACE on a combined lactation basis**
  - **If computing requirements are a limiting factor= compromise reasonable between precision and feasibility**

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